GATE 2006

Sample Test Paper : Mathematics

[EC / EE / IN / CS / IT / ME]

Duration : 60 Min. ***Maximum Marks : 50***

Read the following instructions carefully

1. The Question Paper has 28 questions. Q 1 – 10 carry one mark each and Q.11 – 28 carries two marks each.
2. Q.27 and Q.28 are interlinked questions of 2 marks each.
3. Attempt all questions.
4. Each question has four choices (A) , (B) , (C) and (D) of which only one choice is correct.
5. Choose the **closest** numerical answer among the choices given.
6. Questions are to be answered by darkening with a soft HB pencil the appropriate bubble (A) , (B) , (C) or (D) against the question number on the ORS available with the paper.
7. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
8. There will be **negative** marking. For each wrong answer 0.25 marks from Q.1 to 10 and 0.5 marks from Q.11 to 26 will be deducted. However Q.27 and Q.28 carry no negative marks. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
9. The Answer Sheet is an Objective Response Sheet (ORS).
10. No charts or tables are provided in the examination hall.
11. Use the blank page given at the end of the question paper for rough work.

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Q1 – Q10 carry one mark each

1. A single letter is selected at random from the word "PROBABILITY". The probability that it is a vowel is

(A) $\frac{1}{3}$ (B) 2 / 21

(C) 0 (D) 2 / 9

2. In a horse race the odds in favour of four horses H_1, H_2, H_3, H_4 are 1 : 3, 1 : 4, 1 : 5; 1 : 6 respectively. Not more than one wins at a time. Then the chance that one of them wins is

(A) 320/419 (B) 319/420

(C) 419/520 (D) 520/519

3. In a garden 40% of the flowers are roses and the rest are carnations. If 25% of the roses and 10% of the carnations are red, the probability that a red flower selected at random is a rose is

(A) 5/6 (B) 1/4

(C) 4/5 (D) 5/8

4. What is the probability of a particular person getting 9 cards of the same suit in one hand at a game of bridge where 13 cards are dealt to a person ?

(A) $\frac{{}^{13}C_9 \times 4}{{}^{39}C_4}$ (B) $\frac{{}^{13}C_9 \times {}^{13}C_9 \times {}^{13}C_2 \times 4}{{}^{52}C_9}$

(C) $\frac{{}^{13}C_9 \times {}^{13}C_4 \times 4}{{}^{52}C_{13}}$ (D) $\frac{{}^{13}C_9 \times 4}{{}^{52}C_{13}}$

5. The no. of students who obtained marks between 40 & 45 is

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

(A) 10 (B) 12

(C) 18 (D) 17

6. The following equations have solutions

$$x + 2y - z = 3$$
$$2x - 2y + 3z = 2$$
$$3x - y + 2z = 1$$
$$x - y + z = -1$$

(A) x = –1, y = 4, z = 4 (B) x = 4, y = 1, z = 1

(C) x = –1, y = 4, z = –4 (D) infinite solutions

7. Consider the system of equations

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

This system has

(A) Unique solution (B) Infinite number of solution
(C) No solution (D) Exactly two solutions

8. The equation $2x^3 - 3x - 45 = 0$ has a root x = a.
The approximate solution of the equation $2.1x^3 - 2.9x - 47 = 0$ is

(A) 1.99 (B) 0.99
(C) −2.98 (D) 2.98

9. Let f(x) = x exp(x) − 1, x ∈ R. Then f(x) = 0 has

(A) no solution in R (B) exactly one solution in R
(C) more than one solution in R (D) none of these

10. Let C be the boundary of the triangle where vertices are (0, 0) $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. Then

$$\oint_c (y - \sin x)\,dx + \cos x\; dy =$$

(A) $-\frac{\pi}{4} - \frac{2}{\pi}$ (B) $\frac{\pi}{4} - \frac{2}{\pi}$
(C) $\frac{2}{\pi} - \frac{\pi}{4}$ (D) $\frac{\pi}{4} + \frac{2}{\pi}$

Q11 – Q16 carry two marks each

11. Ten points are marked on a straight line and 11 points are marked on another straight line. How many triangles can be constructed with vertices from among the above points? And we choose any three points from all the given points, what is the probability that it forms a triangle?

(A) 1045, $\frac{{}^{21}C_3}{\left({}^{10}C_3 + {}^{11}C_3\right)}$ (B) 1045, $\frac{11}{14}$
(C) 495, $\frac{{}^{21}C_3}{\left({}^{10}C_3 + {}^{11}C_3\right)}$ (D) 495, $\frac{11}{14}$

12. A man has 9 friends (4 men and 5 women). If there have to be exactly 3 women in the invitees, what is the probability that the invitees have two men ?

(A) $\frac{13}{16}$ (B) $\frac{12}{16}$
(C) $\frac{11}{16}$ (D) $\frac{9}{16}$

13. A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident ?

(A) 75% (B) 80%
(C) 35% (D) 100%

14. The chances of a person being alive who is now 35 years old, till he is 75 are 8 : 6 and of another person being alive now 40 years old till he is 80 are 4 : 5. The probability that at least one of these persons would die before completing 40 years hence is

(A) 8/14 (B) 16/63
(C) 4/9 (D) 47/63

15. Two cards are drawn with replacement from a well shuffled deck of 52 cards. The mean and standard deviation for the number of aces are

X	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Probability distribution table

(A) $\frac{2}{13}, 0.377$ (B) $\frac{1}{13}, 0.277$
(C) $\frac{3}{13}, 0.477$ (D) none of the above

16. There are 64 beds in a garden and 3 seeds of a particular type of flower are shown in each bed. The probability of a flower being white is 1/4. The number of beds with 3, 2, 1, and 0 white flowers is respectively

(A) 1, 9, 27, 27 (B) 27, 9, 1, 27
(C) 27, 9, 1, 1 (D) 27, 9, 9, 1

17. Six dice are thrown 729 times. How many times do you expect atleast three dice to show a five or a six.

(A) 244 (B) 234
(C) 232 (D) 233

18. Sum to n terms of 1.3.5 + 3.5.7 + 5.7.9 + ….. is

(A) $2n^3 + 8n^2$ (B) $n[2n^3 + 8n^2 + 7n - 2]$
(C) $(n + 1)[2n^3 + 8n^2 + 7n - 2]$ (D) $4n[2n^3 + 7n - 2]$

19. A root of the equation $f(x) \equiv x \sin x - 1 = 0$ lies in the interval [0, 2]. Taking a = 0, b = 2 and using the secant method, at the end of third iteration, the approximate value of the root will be

(A) 1.1212 (B) 1.1141
(C) 1.0997 (D) None of these

20. Let x = r be a simple root of the equation f(x) = 0 such that $r \in [a, b]$. Let c_0, c_1, c_2, c_n, ….. be a sequence generated by the bisection method to locate the root of f(x) in [a, b]. Then $|r - c_n| \leq$

(A) $\frac{|b-a|}{2^{n+1}}$ (B) $\frac{|b-a|}{2^{n-1}}$

(C) $\frac{|b-a|}{2^{n+3}}$ (D) None of these

21. Let $I = \int_0^{\pi/2} \sqrt{\sin x}\, dx$. Taking 8 intervals and using Simpson's 1/3 rule, I will be approximately equal to the values (of sqrtf (sin x)) are given in the table below:

0	0
0.261799	0.508743
0.523599	0.707107
0.785398	0.840896
1.047198	0.930605
1.308997	0.982815
1.570796	1

(A) 1.38728 (B) 1.18728

(C) 1.18728 (D) None of these

22. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then value of A^n is

(A) $\begin{bmatrix} n\cos\alpha & n\sin\alpha \\ -n\sin\alpha & n\cos\alpha \end{bmatrix}$ (B) $\begin{bmatrix} -\cos n\alpha & -\sin n\alpha \\ \sin n\alpha & -\cos n\alpha \end{bmatrix}$

(C) $\begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$ (D) $\begin{bmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$

23. The value of determinant of matrix $\begin{bmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{bmatrix}$ is

(A) $a^3 . b^3 .c^3 . d^3 - abcd$ (B) $a^2 b^2 c^2 d^2 - abcd$

(C) 0 (D) 1

24. T_p, T_q, T_r are the p^{th}, q^{th} & r^{th} terms of an A.P. then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(A) 1 (B) −1
(C) 0 (D) p + q + r

25. The elevation of land above sea level, h, at a point (x, y) is given by

$$h(x, y) = \exp\left[-0.01(x^2 + y^2)\right]$$

A car travels through the terrain, so that its coordinates x and y depend on time in the following way :

$$x(t) = -7 + 10\cos(10t), \quad y(t) = 4 + 10\sin(10t)$$

The speed with which the altitude of the car increases / decreases at t = 0 is

(A) $-8\exp(-0.25)$ (B) $8\exp(-0.25)$
(C) $-16\exp(-0.25)$ (D) $16\exp(-0.25)$

26. Let a surface S in three dimensions be defined by $z = xy + x^2$. Then the slope of the surface at P(2, 3) in the direction $\theta = -120°$ is

(A) $\frac{7+2\sqrt{3}}{2}$ (B) $\frac{-7+2\sqrt{3}}{2}$
(C) $\frac{-7-2\sqrt{3}}{2}$ (D) $\frac{7-2\sqrt{3}}{2}$

Q27(a), (b) & Q28(a), (b) carry two marks each

Interlinked Question :

27(a). Let f(x) be a continuous function on the interval [a, b] such that $a \le f(x) \le b$ for $a \le x \le b$. Then

(A) ∃ no number $c \in [a, b]$ such that f(c) = c
(B) ∃ exactly one number $c \in [a, b]$ such that f(c) = c
(C) ∃ more than one number $c \in [a, b]$ such that f(c) = c
(D) None of these

27(b). Further let $|f'(x)| < 1$ for $a \le x \le b$. Then

(A) There exists exactly one number $c \in [a, b]$ such that f(c) = c
(B) There exists no number $c \in [a, b]$, such that f(c) = c
(C) There exists infinite number of numbers $\in [a, b]$, such that f(c) = c
(D) None of these

28(a). Let the vector field be given by $\overline{F} = xy\,\hat{i} + y^2\,\hat{j}$. Then the counterclockwise circulation of $\overline{F}$ around and over the boundary of the region enclosed by the curves $y = x^2$ and y = x in the first quadrant is

(A) $-\frac{1}{12}$ (B) $-\frac{1}{6}$

(C) $-\frac{1}{3}$ (D) $\frac{1}{12}$

28(b). Further, the outward flux of $\overline{F}$ across C =

(A) $-\frac{1}{5}$ (B) $\frac{1}{5}$

(C) –1 (D) none of these

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